Section 5.3 Partial Fractions

Wednesday, April 22, 2020 9:01 AM

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{9}=\frac{9}{18}+\frac{6}{18}+\frac{2}{18}=\frac{17}{18}=\frac{17}{2\cdot 3^2}=\frac{A}{2}+\frac{B}{3}+\frac{C}{3^2}$$

From Section 5.2

$$\frac{12}{s(s+4)}=\frac{A}{s}+\frac{B}{s+4}$$

$$12=A(s+4)+Bs$$

Method 1: systems of equations

$$12=As+4A+Bs$$

$$A+B=0$$
$$4A=12$$
$$A=3$$
$$B=-3$$

Method 2: numbers.

$s=-4$: $12=B(-4)$, $-3=B$

$s=0$: $12=4A$, $3=A$

$$\frac{12}{s(s+4)}=\frac{3}{s}-\frac{3}{s+4}$$

Ex: quadratic factor

$$f(x)=\frac{2x^2+x-8}{x^3+4x}=\frac{2x^2+x-8}{x(x^2+4)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}$$

$$2x^2+x-8=A(x^2+4)+(Bx+C)x$$

let $x=0$: $-8=4A$

$$-2=A$$

$$2x^2+x-8=-2x^2+8+Bx^2+Cx$$

$-2 = A$

$2x^2 + x - 8 = -2x^2 - 8 + Bx^2 + Cx$

$C = 1$

$2x^2 = (-2 + B)x^2$

$2 = -2 + B$

$4 = B$

$$f(x) = \frac{-2}{x} + \frac{4x + 1}{x^2 + 4}$$

Ex: $$\frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$ $\cdot (x+3)(x-1)^2$

Repeated factors.

$$3x^2 - 8x + 13 = A(x-1)^2 + B(x-1)(x+3) + C(x+3)$$

$x = 1$ $\quad 3 - 8 + 13 = 4C$

$8 = 4C$

$2 = C$

$x = -3$ $\quad 3(9) - 8(-3) + 13 = A(-4)^2$

$64 = 16A$

$4 = A$

$3x^2 - 8x + 13 = 4(x-1)^2 + B(x-1)(x+3) + 2(x+3)$

$x = 0$ $\quad 13 = 4 + B(-1)(3) + 2(3)$

$3 = -3B$

$-1 = B$

Ex: $\mathcal{L}^{-1}\left\{\frac{s^2 + 4}{s^4 - s^2}\right\}$

Ex: $\mathcal{L}^{-1}\left\{\dfrac{s^2+1}{s^4-s^2}\right\}$

$$\frac{s^2+4}{s^2(s^2-1)} = \frac{s^2+4}{s^2(s-1)(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{s+1}$$

$0 \quad -4 \quad +5/2 \quad -5/2$

$$s^2+4 = A\,s(s-1)(s+1) + B(s-1)(s+1) + C\,s^2(s+1) + D\,s^2(s-1)$$

$-4 \quad +5/2 \quad -5/2$

$s=0 \qquad 4 = -B \Rightarrow B=-4$

$s=1 \qquad 5 = 2C \Rightarrow C = 5/2$

$s=-1 \qquad 5 = -2D \Rightarrow D = -\frac{5}{2}$

$s=2 \qquad 8 = A(2)(1)(3) - 4(1)(3) + \frac{5}{2}(4)(3) - \frac{5}{2}(4)(1)$

$$8 = 6A - 12 + 30 - 10$$

$$0 = 6A$$

$$0 = A$$

$$\mathcal{L}^{-1}\left\{\frac{-4}{s^2} + \frac{5}{2}\,\frac{1}{s-1} - \frac{5}{2}\,\frac{1}{s+1}\right\}$$

$\alpha=1 \qquad \alpha=-1$

row 3 $\mathcal{L}^{-1}\left\{\dfrac{n!}{s^{n+1}}\right\} = t^n$ $\qquad -4\,\dfrac{1}{s^2}$

row 4 $\mathcal{L}^{-1}\left\{\dfrac{1}{s-\alpha}\right\} = e^{\alpha t}$

$$= -4t^1 + \frac{5}{2}e^{t} - \frac{5}{2}e^{-t}$$